

MATH 306 Workshop

Theorem 1.34: Conditions for a subspace

Theorem 1.44: Condition for a direct sum

Theorem 1.45: Direct sum of two subspaces

Theorem 2.21: Linear Dependence Lemma

Theorem 2.23: Length of linearly independent list \leq length of a spanning list

Theorem 2.29: Criterion for basis

Theorem 2.31: Spanning list contain a basis

Theorem 2.32: Every finite dimensional vector space has a basis

Theorem 2.33: Linear independent list extends to a basis

Theorem 2.34: Every subspace of V is part of a direct sum equal to V

Theorem 2.39: Linear independent list of the right length is a basis

Theorem 2.42: Spanning list of the right length is a basis

Theorem 2.43: Dimension of a sum

Theorem 3.14: The null space is a subspace

Theorem 3.16: Injectivity is equivalent to null space equals $\{0\}$

Theorem 3.19: The range is a subspace

Theorem 3.22: Fundamental Theorem of Linear Map

Theorem 3.23: A map to a smaller dimension space is not injective

Theorem 3.24: A map to a larger dimension space is not surjective

Theorem 3.54: Inverse of an invertible linear map is unique

Theorem 3.56: Invertibility is equivalent to injectivity and surjectivity

Theorem 3.59: Dimension shows whether vector spaces are isomorphic

Theorem 3.69: Injectivity is equivalent to surjectivity in finite dimensions

Definition 5.2: Invariant subspace

Definition 5.5: Eigenvalue

Definition 5.7: Eigenvector

Theorem 5.10: Linear independent eigenvectors

Theorem 5.21: Operators on F. D. N. T. C. V. S. have an eigenvalue

Theorem 5.27: Every operator has an upper-triangular matrix

Theorem 5.30: Determination of invertibility from upper-triangular matrix

Theorem 5.32: Determination of eigenvalue from upper-triangular matrix

Definition 5.39: Diagonalizable

Theorem 5.41: Conditions equivalent to diagonalizability

More practices:

1. Let A be a n by n invertible matrix that satisfies $A^{-1}=A$. What are all the possible eigenvalues of A ?

2. Let V be a vector space. Assume S and T are invertible operators on V .

Prove that $(TS)^{-1} = S^{-1}T^{-1}$.

3. Suppose $T \in \mathcal{L}(V)$ and there exists a basis B for V such that

$$\mathcal{M}(T, B) = \begin{pmatrix} 1 & 7 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 8 & 2 \\ 0 & 0 & 0 & 3 \end{pmatrix}, \text{ is } T \text{ diagonalizable? Explain}$$